

Carbon taxation and precautionary savings*

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Abstract

This paper asks how precautionary savings affect the level of the optimal carbon tax. I augment a heterogeneous-agent incomplete-markets model with a climate sector and estimate its structural parameters with indirect inference. As households in the model engage in precautionary saving behavior, it replicates a stylized fact from the data that the marginal propensity to consume pollution-intensive goods decreases with income. Therefore, the carbon tax and the redistribution of its revenue have distributional consequences. When recycling the revenue lump-sum, the optimal carbon tax also serves as an insurance device for the uninsurable idiosyncratic productivity shocks. As a consequence, the optimal tax is higher than what is required to internalize the negative climate externality.

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1 Introduction

A carbon tax on polluting economic activity is seen as the most efficient way to tackle anthropogenic global warming¹, "the greatest market failure the world has ever seen" (Stern, 2007). Economic insights about the optimal *level* of the carbon tax stem predominantly from models where households can perfectly insure shocks to their labor income. Hence, realistic market features such as borrowing constraints and uninsurable idiosyncratic risk play no role in these models. Yet, from the quantitative macroeconomics literature we know that these features affect household demand in non-trivial ways, as agents accumulate precautionary savings.² As a result, the optimal carbon tax - in addition to the climate externality - might also take into account distributional and insurance concerns. The goal of the present paper is thus to investigate how and to what extent the optimal level of the carbon tax changes when precautionary saving behavior is taken into account.

To do so, I augment a heterogeneous-agent incomplete-markets model with a climate sector, and estimate its structural parameters with indirect inference. In particular, the demand side of the model, where households consume a clean and a pollution-intensive (dirty) good, captures two salient stylized facts. Both have important implications for the optimal carbon tax. First, the marginal propensity to consume dirty goods, which I henceforth term *marginal propensity to pollute* (MPPs), is decreasing in income (Weber and Matthews, 2008; Levinson and O'Brien, 2019; Sager, 2019). This heterogeneity in MPPs, so far ignored in the quantitative literature, arises naturally from my model with incomplete markets. The key is that agents engage in precautionary saving against future idiosyncratic risk that renders their consumption function concave in current income and wealth (Carroll and Kimball, 1996; Huggett and Ospina, 2001). Indeed, as my main theoretical result, I show that Engel curves - which describe the relation between *income* and consumption - are concave under quasi-homothetic utility over both goods and uninsurable productivity shocks.

This is important because, *ceteris paribus*, decreasing MPPs (or concave Engel curves) give rise to a so-called "equity-pollution dilemma" (Sager, 2019) such that progressive redistribution of income increases aggregate pollution. This feedback might call for a lower carbon tax. However, higher lump-sum transfers through recycled carbon revenue contributes to insuring household risk and thus might call for a higher carbon tax. In addition to these general equilibrium and distributional arguments, Jacobs and van der Ploeg (2019) demonstrate that the shape of Engel curves itself matters for optimal corrective taxation in static settings.

To get an intuitive understanding for the concavity result, note that the dynamic problem of the household can be divided into two steps. First, she decides how much to save for the next period and how much to spend on consumption. Second, given the amount of total expenditure, the household decides how much to spend on the dirty and the clean good, respectively. This latter problem is void of any dynamic decisions and hence, static in nature. Finally, since in the static

¹ The "Economists' Statement on Carbon Dividends" has been signed by over 3500 economists, including 45 nobel laureates, and, among other things, states that "A carbon tax offers the most cost-effective lever to reduce carbon emissions at the scale and speed that is necessary." (www.econstatement.org)

² In recent papers, for instance, Holm (2018) and Carroll, Holm and Kimball (2021) provide analytical insights about the interaction between precautionary saving and borrowing constraints.

problem quasi-homothetic preferences are used, the decision rule from total expenditure to clean or dirty consumption is linear. However, total expenditure is concave in current assets and income and hence, consumption in clean or dirty goods is also concave.

Turning to the second stylized fact, poorer households spend a larger fraction of their income on dirty goods. I follow the literature and model this by introducing a subsistence level of dirty goods consumption in household preferences. Again, this implies a trade-off. On the one hand, taxing the dirty consumption good is very efficient as the subsistence level lowers the elasticity of substitution between the clean and the dirty good. On the other hand, the incidence of the carbon tax is now regressive.³

The supply side of the model follows [Barrage \(2020\)](#) and is specified as follow. A final goods firm uses capital, labor, and energy as inputs to production, while energy in turn is produced by a second firm using capital and labor only. Importantly, energy production is pollution intensive and entails a climate externality: it increases the stock of carbon in the atmosphere which in turn decreases economic productivity due to a damage function ([Nordhaus, 1993](#); [Golosov, Hassler, Krusell and Tsyvinski, 2014](#)). Finally, a government levies labor income, capital income, and carbon taxes to finance interest payment on bonds and lump-sum transfers. Throughout, the government keeps labor income and capital income taxes fixed and chooses the lump-sum transfer to balance its budget constraint.

The main computational exercise then proceeds in two steps. First, I calibrate and estimate a subset of the model's structural parameters using simulation methods and U.S. data. In my estimation, I use a comprehensive U.S. household panel data set - the Panel Study of Income Dynamics (PSID) - which includes information on demographics, income, wealth, and consumption. Second, I compute the the level of the carbon tax which yields the highest (utilitarian) welfare in the steady-state of the economy when carbon revenue is recycled lump-sum. Lastly, I study how this welfare-maximizing tax changes in economic environments without borrowing constraints and idiosyncratic risk.

In the first step, I externally calibrate part of the parameter vector, including labor income and capital income taxes, to match features of the U.S economy. Moreover, using the PSID data, I estimate the stochastic productivity process on (imputed) wage data from the PSID with a minimum distance estimation. A key input to capture the quantitative relevance of incomplete markets. Lastly, I estimate the remaining structural parameters of the model using a mixture of simulated method of moments and indirect inference as in [Guvenen and Smith, Jr. \(2014\)](#) and more recently in [Stoltenberg and Uhlenhorff \(2022\)](#). In particular, I propose a novel strategy to estimate the subsistence level and relative preference of dirty goods in which I use the household decision rule in the static framework as my auxiliary model. The parameters are precisely estimated and I provide evidence for global identification.

In the second step, I use my estimated model as a laboratory to study the effects of precautionary savings on the optimal carbon tax. I compare my results to a complete markets (CM) version

³ See [Hummel and Ziesemer \(2021\)](#) for a similar reasoning.

of the model in which I get rid of idiosyncratic risk and replace the no-borrowing constraint by the natural debt limit. Since the wealth distribution is indeterminate in the steady-state of this economy (Chatterjee, 1994), I follow Dyrda and Pedroni (2022) and impose the invariant distribution to be the same as in the incomplete markets model (IM) under the optimal tax. The idea is that if precautionary savings did not matter, the optimal carbon tax should not change under the CM specification.

As my main result, I find that the carbon tax (as a fraction of the price of the dirty good) is more than five times higher under the precautionary saving case. Under this third-best policy setting, that is, keeping other taxes fixed, the main quantitative mechanism is that the carbon tax generates revenue which is recycled lump-sum and provides insurance for households. This stark increase in the optimal level of the carbon tax indicates that the welfare cost of not optimizing of the other instruments is very big. Intuitively, the government loads all its tasks, that is, raising revenue, internalizing the climate externality, and redistribution, on the carbon tax.

When increasing the curvature of the consumption function - and thus the precautionary motive - by varying the degree of prudence under risk, the carbon tax decreases. This change suggests that the insurance motive of the planner is weakened, as self-insurance of households increases as absolute prudence increases. In addition, this is suggestive evidence for the equity-pollution dilemma under a given level of idiosyncratic risk.

Related literature and contribution My paper contributes to several strands of the literature overarching optimal fiscal policy, consumption dynamics, and environmental economics.

My key contribution to this literature is the *joint analysis* of optimal carbon taxation in an environment with idiosyncratic risk which generates precautionary savings. The quantitative model combines a heterogeneous-agent incomplete market economy in the spirit of Bewley (1986); Huggett (1993); Aiyagari (1994) with a climate sector, which yields an endogenous distribution over income and wealth, and heterogeneity in marginal propensities to consume. Hence, my model allows to study the interaction of climate policies and economic inequalities in a unified framework. Thereby, I connect two lines of research.

The first line is a rapidly growing literature which analyzes optimal carbon taxation in quantitative macroeconomic models. Building on the seminal work by Nordhaus (1992, 1993), who developed the first integrated assessment model (IAM) to analyze climate damages within a centralized economic framework, several papers moved to decentralized market structure. For instance, Golosov et al. (2014) derive a formula for the optimal carbon tax in a dynamic stochastic general-equilibrium model with an externality and resource scarcity. Building on their quantitative work, Barrage (2020) quantifies optimal carbon taxation in a model with tax distortions, and in turn, Douenne, Hummel and Pedroni (2022) quantify the additional impact of inequality.⁴ None of these papers investigate settings with idiosyncratic risk, which is my main contribution compared

⁴ Other quantitative examples study the optimal environmental policy in response to business cycles (Heutel, 2012) or nominal frictions and uncertainty (Annicchiarico and Di Dio, 2015), or in an overlapping generations framework (Kotlikoff, Kubler, Polbin and Scheidegger, 2021a; Kotlikoff, Kubler, Polbin, Sachs and Scheidegger, 2021b).

to existing frameworks.⁵

The second line investigates the impact of idiosyncratic uncertainty and borrowing constraints on individual consumption demand. In particular, in the presence of idiosyncratic risk both prudence in preferences as well as borrowing constraints give rise to a precautionary saving motive which renders the consumption function concave in current income and wealth (Leland, 1968; Sandmo, 1970; Zeldes, 1989^{b,a}; Kimball, 1990^{a,b}; Carroll and Kimball, 1996; Huggett and Ospina, 2001; Carroll et al., 2021).⁶

Second, my exercise builds on the theoretical literature on optimal carbon taxation. In particular, Jacobs and van der Ploeg (2019) show that the optimal carbon tax should be equal to the marginal external damage of pollution if Engel curves are linear and the social planner has access to a non-individual lump-sum transfer and linear income taxes.⁷ In other words, the optimal carbon tax follows the Pigouvian rule (Pigou, 1920).⁸ Intuitively, any demand change induced by the carbon tax can be undone by changing the lump-sum transfer and the income tax. The main difference in this paper is that I consider a quantitative model with idiosyncratic risk, CRRA utility, and borrowing constraints in which, as explained above, non-linear Engel curves are microfounded.⁹

Recent studies further extend these theoretical analyses under deterministic/non-stochastic environments with tax distortions (Barrage, 2020) and inequality (Douenne et al., 2022). Compared to this theoretical literature I do not have analytical results concerning the optimal carbon tax, because a closed-form solution is not obtainable within the class of models I consider. Instead, I conduct counterfactual analyses to disentangle the main forces behind my results, as is common in this literature (see e.g. Conesa, Kitao and Krueger, 2009; Dyrda and Pedroni, 2022).

In addition, my paper relates to the literature on subsistence consumption of carbon-intensive goods and the incidence of taxation (Klenert and Mattauch, 2016; Klenert, Schwerhoff, Edenhofer and Mattauch, 2018). I contribute to this literature by proposing a novel strategy to estimate the structural parameters - including the subsistence level of dirty goods consumption - of the model via indirect inference (Guvenen and Smith, 2014; Stoltenberg and Uhlendorff, 2022).¹⁰

Lastly, my paper builds on the literature which studies how to optimally recycle carbon tax revenue (Fried, Novan and Peterman, 2018, 2021; Goulder, Hafstead, Kim and Long, 2019). This

⁵ An exception is Benmir and Roman (2022) who study the 2050 net zero emissions target for the U.S. in a HANK model. The main difference to the present paper is that I focus on the optimal carbon tax and model household consumption with quasi-homothetic preferences and two goods.

⁶ Lugilde, Bande and Riveiro (2019) survey the empirical literature on precautionary savings. They conclude that papers which "test the effect of uncertainty about future income on consumption/saving decisions, especially [those] using micro data, tend to provide robust and convincing results as regards the existence of a precautionary motive for saving" (p.507). Examples of micro-panel studies in different countries include Carroll and Samwick (1997, 1998); Guariglia and Rossi (2002); Guariglia (2003); Lugilde, Bande and Riveiro (2018).

⁷ This result is reminiscent of earlier studies by Angus Deaton (Deaton, 1979, 1981) in which he demonstrates that uniform commodity taxation is desirable under linear Engel curves and separability in consumption and leisure.

⁸ This refers to Proposition 2 in Jacobs and van der Ploeg (2019).

⁹ Jacobs and van der Ploeg (2019) is a specific application of a more general result that the optimal carbon tax equals the Pigouvian rate adjusted by the marginal cost of public funds (Sandmo, 1975; Bovenberg and van der Ploeg, 1994), which equals one under the optimal tax system (Jacobs and de Mooij, 2015; Jacobs, 2018).

¹⁰ The references in the text refer to recent applications of indirect inference to dynamic macroeconomic models. A theoretical treatment can be found in Gourieroux, Monfort and Renault (1993) and Smith, Jr. (1993).

paper, on the other hand, examines the optimal level of the carbon tax, and the method of revenue recycling is, for now, set to lump-sum transfers.

The article is organized as follows. Section 2 describes the quantitative model. Section 3 presents the data and outlines the estimation strategy. Section 4 shows the estimation results and discusses identification. Section 5 briefly discusses the main computational exercise. Section 6 presents the results.

2 An Economy with Two Goods and Quasi-Homothetic Preferences

This section presents an [Aiyagari \(1994\)](#) economy with a clean and a dirty good, and endogenous labor supply. The structure of production and the climate sector largely follows [Barrage \(2020\)](#) and [Golosov et al. \(2014\)](#), respectively. In this setting, the first-best is not attainable, as the government does not have access to individualized lump-sum transfers.

A key notion will be that the household problem in this more elaborate setting can be broken into two steps. First, the household chooses how much to save and how much to spend. Second, once total spending has been determined, the household decides in a static subproblem how much to spend on the clean and how much to spend on the dirty good.

2.1 Setup

Households Time is discrete, $t \in \{0, 1, \dots\}$, and there is no aggregate risk. The time period in the model is five years. The economy is populated by a continuum of infinitely-lived households of measure one. Households' preferences are represented by the utility function

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_{it}, d_{it}, n_{it}) \quad (1)$$

where c_{it} denotes the consumption flow of the clean good, d_{it} denotes the consumption flow of the dirty good, and n_{it} denotes labor supply of household i at time t . The future is discounted with factor β .

Households are subject to idiosyncratic productivity risk captured by a first-order Markov chain $\theta_t \in \Theta$ with $|\Theta| = S < \infty$ and transition matrix $\Gamma_{S \times S}$. An agents' pre-tax income is then determined by her productivity, the equilibrium wage per unit of productivity, w_t , and the amount of labor supply: $y_t^{pre} = w_t \theta_t n_t$. Pre-tax income is transformed into net (or after-tax) income using a net income function $\mathcal{T}(y) = y - T^y(y)$, where the tax function $T^y(\cdot)$ is to be specified below. Moreover, households have access to a one-period risk-free bond, a , as consumption insurance instrument. Capital income is taxed at rate τ^k and borrowing is restricted by an ad-hoc constraint \underline{a} . Lastly, share $(1 - \mu)$ of energy produced is dirty and hence potentially subject to a carbon tax τ_d , which the energy producer passes-through at rate ω . The government pays lump-sum transfers g to the household.

Hence, the household budget constraint is

$$c_t + (p_d + (1 - \mu)\omega\tau_d)d_t + a_{t+1} = \mathcal{T}(y^{pre}) + (1 + r(1 - \tau^k))a_t + g,$$

where p_d denotes the price of the dirty good, respectively, r is the equilibrium interest rate.

Production I follow the literature and model two production sectors (Golosov et al., 2014; Barrage, 2020; Douenne et al., 2022).

Final good sector In the final goods sector, indexed by 1, a final good Y is produced using a neoclassical aggregate production function

$$Y = (1 - \mathfrak{D}(S))\tilde{X}\tilde{F}_1(K_1, L_1, E^p) = X(S)\tilde{F}_1(K_1, L_1, E^p) = F_1(K_1, L_1, E^p; \tilde{X}, S) \quad (2)$$

with K_1 units of capital, L_1 efficiency units of labor, E^p units of energy as inputs, and total factor productivity \tilde{X} . The final good can either be consumed or invested. $\mathfrak{D}(S)$ represents climate damages to output as a function of the stock of atmospheric carbon S with $\mathfrak{D}'(S) > 0$. This modelling approach of climate damages follows the seminal work by Nordhaus (1991) and the more recent environmental macroeconomic literature.

Energy sector In the energy sector, indexed by 2, energy E is produced using a neoclassical aggregate production function

$$E = F_2(K_2, L_2) \quad (3)$$

with K_2 units of capital and L_2 efficiency units of labor. Energy is either consumed by households (dirty good) or used in production of the final good such that $E = E^p + D$. Following Barrage (2020), producers can provide a share μ from clean energy production, such that only $E^m = (1 - \mu)E$ contributes to the stock of emissions. This clean technology is available at a cost of $\Psi(\mu)$ per unit of energy.

Lastly, capital and labor are fully mobile across sectors such that market clearing implies:

$$K = K_1 + K_2 \quad (4)$$

$$L = L_1 + L_2 \quad (5)$$

Government The government levies labor taxes on pre-tax income y^{pre} using the possibly non-linear labor tax function $T^y(y^{pre})$, a linear capital income tax τ^k as well as a carbon tax on dirty goods consumption τ_d . Moreover, it issues government debt B , and chooses lump-sum transfers g to balance its budget:

$$B_{t+1} + g_t = (1 + r)B_t + \mathfrak{T}_t, \quad (6)$$

where \mathfrak{T}_t denotes total tax revenue from labor, capital, and carbon taxes.

Climate sector

Carbon cycle The current level of atmospheric carbon concentration, S_t , depends on current and past emissions. In my case, emissions are related to energy produced net of the abated share:

$$S_t = \sum_{\tau=0}^{\infty} (1 - \Phi_{\tau}) [(1 - \mu_{t-\tau})E_{t-\tau}] = \sum_{\tau=0}^{\infty} (1 - \Phi_{\tau}) E_{t-\tau}^m$$

where $1 - \Phi_{\tau} = \varphi_L + (1 - \varphi_L)\varphi_0(1 - \varphi)^{\tau}$ with the following interpretation: φ_L is the share of carbon emitted which stays in the atmosphere forever; a share of $1 - \varphi_0$ of the remaining $1 - \varphi_L$ exits the atmosphere immediately; and a remaining share $(1 - \varphi_L)\varphi_0$ that decays at geometric rate φ . To write it recursively, following [Känzig \(2021\)](#), I set and $\varphi_L = 0$ and write

$$S_t = (1 - \varphi)S_{t-1} + \varphi_0 E_t^m \quad (7)$$

Recursive problem An agent is characterized by the by the pair $(a_{it} = a, \theta_{it} = \theta)$, the household state, and solves the following optimization problem

$$\begin{aligned} V(a, \theta) &= \max_{c, d, n, a'} u(c, d, n) + \beta \mathbb{E}_{\theta} V(a', \theta') \\ &\text{subject to} \\ c + (p_d + (1 - \mu)\tau_d \omega)d + a' &\leq (1 + r(1 - \tau^k))a + \underbrace{w\theta n - T^y(w\theta n)}_{T(w\theta n)} + g \\ a' &\geq \underline{a} \end{aligned} \quad (8)$$

2.2 Equilibrium

Let $A \equiv [a, \bar{a}]$ be the set of possible values for a_{it} . Define the state space by $\mathbb{S} \equiv A \times \Theta$ and let the σ -algebra $\Sigma_{\mathbb{S}}$ be defined as $B_A \otimes P(\Theta)$, where B_A is the Borel σ -algebra on A and $P(\Theta)$ is the power set of Θ . Finally, let $\mathcal{S} = (\mathcal{A} \times \Theta)$ denote a typical subset of $\Sigma_{\mathbb{S}}$. I define a steady-state equilibrium as follows.

Definition 1 (Steady-state equilibrium). A steady-state equilibrium is a government policy $\{\tau_d, g\}$, a vector of aggregate quantities $\{Y, K_1, K_2, L_1, L_2, \mu, E, S\}$, a probability measure Λ defined over the measurable space $(\mathbb{S}, \Sigma_{\mathbb{S}})$, a set of policy functions $\{c(a, \theta), d(a, \theta), n(a, \theta), a'(a, \theta)\}$, a set of prices $\{r, w, p_d\}$, and a set of policies $\{g, \tau_0, \tau_1, \tau_2, \tau^d, \tau_d\}$ such that: (i) given policies and prices, the decision rules solve the optimization problem Equation (8), (ii) the final goods firm chooses capital K_1 , labor in efficiency units L_1 , and energy E^P to maximize profits, (iii) the energy producer chooses capital K_2 , labor in efficiency units L_2 , and abatement μ to maximize profits, (iv) the government budget constraint

$$g + rB = \int_{(A \times \Theta)} T^y(w\theta n(a, \theta)) d\Lambda + \tau^k rA + \tau_d(1 - \mu)E$$

holds, (v) the asset market clears

$$A \equiv \int_{(A \times \Theta)} \mathbf{a}'(a, \theta) d\Lambda = B + K$$

(vi) the goods market clears¹¹

$$\int_{(A \times \Theta)} \mathbf{c}(a, \theta) d\Lambda + \delta K + \Psi(\mu)E = Y,$$

(vii) Λ is an invariant probability measure and satisfies for all $S \in \Sigma_S$

$$\Lambda(S) = \int_{(A \times \Theta)} Q((a, \theta), S) d\Lambda,$$

where Q is the associated Markov transition function induced by Γ and \mathbf{a}' , and (viii) the stock of emissions stays constant at $S = \frac{\varphi_0}{\varphi}(1 - \mu)E$.

2.3 Quasi-homothetic preferences

This section brings forward the functional form assumption regarding the flow utility $u(\cdot)$ in Equation (1). In particular, the two goods setup is cast in an environmental context where one good plays the role of a *clean* good, and the other the role of the *dirty* good. The latter good is subject to a subsistence requirement and in the literature often interpreted as food, fuel, or electricity consumption.

I assume that the utility function is separable in the consumption composite $\tilde{c} = c^\eta(d - \underline{d})^{1-\eta}$ and labor, which precludes non-uniform commodity taxation due to leisure complementarities (Corlett and Hague, 1953). I normalize the time endowment to 1 to specify

$$u(c, d, n) = \frac{(c^\eta(d - \underline{d})^{1-\eta})^{1-\gamma}}{1 - \gamma} + \chi \frac{(1 - n)^{1-\epsilon}}{1 - \epsilon}. \quad (9)$$

The first part of Equation (9) nests Stone-Geary utility in a CRRA specification.¹² In particular, γ denotes the coefficient of relative risk aversion and \underline{d} is the subsistence level for the dirty consumption goods. It is important to note that the elasticity of substitution between the clean and the dirty good is decreasing in the subsistence level (Baumgärtner, Drupp and Quaas, 2017).¹³ η and $(1 - \eta)$ are expenditure shares based on total income net subsistence consumption, as will become clear below. Regarding the second part, χ denotes the disutility of labor supply, and ϵ is related to the Frisch elasticity of labor supply, $\frac{1}{\epsilon} \frac{1-n}{n}$.

The reason for choosing this particular form of preferences - as already indicated above in the stylized model - is that empirical evidence shows a declining expenditure share of pollution intensive goods (dirty goods) with respect to expenditures or income. Key references include Levinson and O'Brien (2019) or Sager (2019) who use the Consumer Expenditure Survey (CEX).

¹¹ This market clearing condition is actually redundant by Walras's law, but is nevertheless a useful check whether all equilibrium conditions are properly computed. Appendix A.3 shows why.

¹² For now I abstract from utility damages of emissions (Bovenberg and van der Ploeg, 1994; Williams, 2002).

¹³ Under no subsistence consumption this elasticity is one (usual Cobb-Douglas case).

This declining pattern is also visible in the PSID, which I will use later for estimation.

Static subproblem To understand the role of the subsistence level, and to facilitate the discussion below, it is instructive to separate the household problem into a dynamic and a static one. In the dynamic problem, the household chooses how much to save for the next period, a_{it+1} , and how much to spend on consumption. Denote this latter total expenditure by e_{it} . In the static problem, the household decides allocates total expenditure between the clean and the dirty good, respectively. Formally, the household solves the following simple problem, in which e_{it} is pre-determined:

$$u(e_{it}) = \max_{c_{it}, d_{it}} c_{it}^\eta (d_{it} - \underline{d})^{1-\eta}$$

subject to:

$$c_{it} + (p_d + \tau_d)d_{it} = e_{it}$$

$$c_{it} \geq 0, \quad d_{it} \geq \underline{d}$$

The solution to this problem is

$$\begin{aligned} c_{it} &= \eta \left(e_{it} - (p_d + \tau_d)\underline{d} \right), \\ d_{it} &= (1 - \eta) \frac{e_{it}}{p_d + \tau_d} + \eta \underline{d}. \end{aligned} \tag{10}$$

The fact that both decision rules are linear in expenditure is an important feature of these particular preferences. In fact, the system of demand equations implied by them are referred to as the Linear Expenditure System (Stone, 1954). In fact, what is required to derive linear decision rules in this static setting are quasi-homothetic preferences.

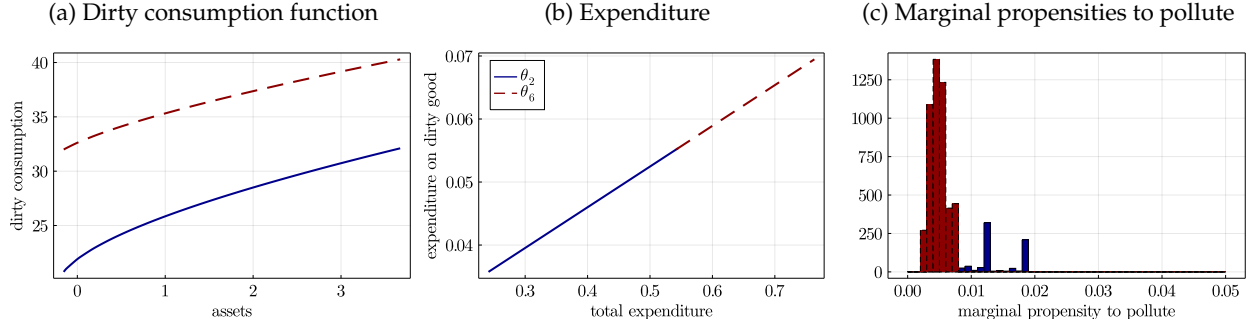
2.4 Concavity of the consumption function(s)

Having presented the model, I want to conclude this section by discussing the household policies in steady state.

Recall again Equation (10) where consumption on either good is linear in expenditure e_{it} . Moreover, Jacobs and van der Ploeg (2019) show, in a static setting where expenditure equals income, that under linear Engel curves, externality correcting taxes should be set at the Pigouvian rate.

It is worth noting, however, that Engel curves are *not* linear under the model described in Section 2.1, even with (CRRA-nested) Stone-Geary preferences. The key to this observation lies in the concavity of the consumption function in heterogeneous-agent incomplete-markets models (Zeldes, 1989b; Carroll and Kimball, 1996). Due to uncertain future income or productivity, households accumulate precautionary savings and especially so when asset and/or income levels are low. Hence, poorer households with relatively more precautionary savings have low consumption and hence, higher marginal propensities to consume. In other words, the Engel curve is non-

Figure 1: Decision rules, expenditure, and marginal propensities to pollute



Note.

linear.¹⁴

Figure 1 illustrates these points. Panel 1a shows consumption functions for two productivity types as a function of assets; both are clearly concave and more so for lower levels of assets. Panel 1b shows expenditure on the dirty good as a function of total expenditure. We see that this relation is linear, relating to the static subproblem of the household (Equation (10)). Panel 1c shows the marginal propensity to consume the dirty good, what I term the *marginal propensity to consume (MPP)*, out of a windfall income gain of 1% of average income. We see that there is a distribution of MPPs, with higher marginal propensities for the lower productivity type. This heterogeneity is a clear indication of non-linear Engel curves.

The following proposition formalizes this discussion:

Proposition 1 (Non-linear Engel curves). *Under (quasi-)homothetic preferences, inelastic labor supply, and for any labor-productivity Markov chain which induces non-negative consumption decisions, both the clean and dirty consumption good exhibits concave Engel curves w.r.t. to income and wealth:*

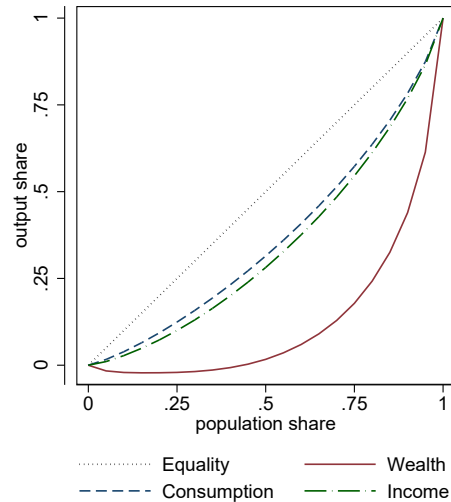
$$c_{aa}(a, \theta) < 0, c_{\theta\theta}(a, \theta) < 0 \quad \text{and} \quad d_{aa}(a, \theta) < 0, d_{\theta\theta}(a, \theta) < 0$$

Proof. The proof of this proposition is a straightforward application of Theorem 1 in Carroll and Kimball (1996) and the fact that the composition of a linear (decision rule in the static problem) and a concave function (decision rule in the dynamic problem) yields a concave function. \square

The key takeaway of Proposition 1 and the preceding discussion is that the optimal carbon tax does take into account distributional concerns in quantitative heterogeneous-agent incomplete market models with precautionary savings.

¹⁴ Carroll and Kimball (1996) discuss two cases under which the consumption function is linear. First, under isoelastic utility and interest rate uncertainty but not idiosyncratic uncertainty. Second, under CARA utility and labor income risk only.

Figure 2: Lorenz curves: Nondurable consumption, net total income, wealth



Note. This figure shows Lorenz curves for nondurable consumption, net total income, and wealth. All variables have been adjusted using the OECD equivalence scale and are expressed in 2010-\$. The respective Lorenz curve expresses how much consumption expenditure (income/wealth) is incurred (earned/held) by the bottom x percent of the population.

3 Quantitative exercise

3.1 Data

I use data from the Panel Study of Income Dynamics between 2005-2018. The PSID is a widely used longitudinal survey containing information on household demographics, income, and wealth. In the waves of 1999 and 2005, respectively, the PSID extended its collection of consumption expenditure data. It now captures over 70 percent of all consumption items available in the Consumer Expenditure Survey (CEX) and around 70 percent of aggregate consumption in the national income and product accounts (NIPA) (Blundell, Pistaferri and Saporta-Eksten, 2016). The PSID was attested to be a high quality dataset in terms of general low sample attrition rates and high response rates (Andreski, Li, Samancioglu and Schoeni, 2014)

Variables The following variables are all on the household level. For instance, income refers to income from both the head and the spouse in the household, if present. Moreover, all monetary variables in the analysis have been adjusted using the OECD equivalence scale and are expressed in 2010-\$.

Income *Labor income* refers to all income from wages, salaries, commissions, bonuses, overtime and the labor part of business income *Total income* in addition includes transfers such as as well as social security income. Both income variables net of taxes, which were computed using NBER's Taxsim program.

Consumption *Nondurable consumption* includes expenses for food, gasoline, rent, utilities, communication, transportation, education, childcare, medical needs, vacations, clothing, and recreational activities. *Total consumption* also includes durable components such as car repair expenses, down-, loan-, and lease-payments for vehicle loans as well as other expenditure regarding vehicles. Moreover, I define *energy expenditure* as expenditure on gasoline, electricity, and heating. All three categories are greenhouse gas intensive goods and are thus used as a data counterpart for the dirty consumption good in the model.

Wealth My wealth variable refers to financial wealth net of liabilities. In particular, I include the value of one's real estate assets net of remaining mortgages, checking and saving accounts, stocks, bonds, business assets, IRAs or other annuities, and cars. I subtract liabilities such as credit card debt, student debt, outstanding medical bills, legal debt, loans obtained from relatives, and business debt.

Sample My baseline sample includes all PSID waves from 2005-2019, and consists of households where the head is between 25 and 60 years. I exclude observations for which information on consumption, income, wealth, education, household size, and region is missing. Furthermore, I remove observations with labor income below half the state minimum wage as well as top and bottom 1% of the remaining observations on consumption, income, and wealth. This leaves me with a sample of 21,750 households, around 2700 observations per year.

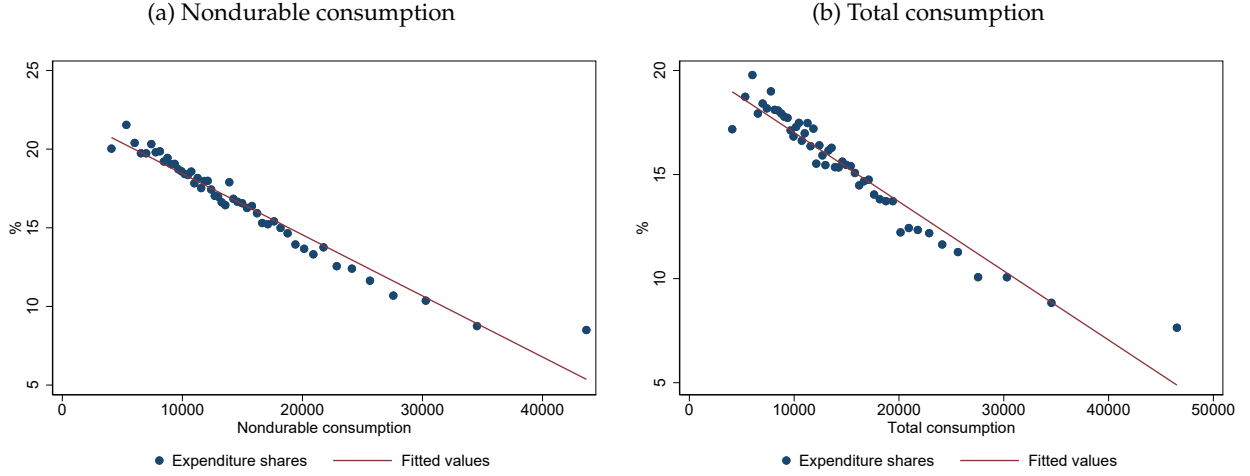
Table D.1 in the appendix shows descriptive statistics about the data. The typical household head is 42 years old, male, and married with 3 family members in total. Figure 2 depicts Lorenz curves for nondurable consumption, income, and wealth respectively.

Average expenditure share $\bar{\epsilon}$ Figure 3 shows energy expenditure — defined as the sum of home fuel, heating, and electricity expenditure — as a share of two different consumption measures in the PSID. The figure shows a clear downward trend from approximately 20% at low consumption expenditure levels to 10% at high levels. As it is well known, a Cobb-Douglas utility function would, assuming equal prices, imply equal expenditure shares for both types of goods irrespective of the expenditure level. Moreover, these expenditure shares are readily observable by the elasticities η and $1 - \eta$. As discussed above, the introduction of the subsistence parameter d , however, implies a consumption allocation with declining expenditure shares of the dirty good as seen in Figure 3.

3.2 Data targets in estimation

Hours worked The cross-sectional average of weekly working hours of household heads in my sample is 40.61. Given that a full week has 168 hours and assuming 8 hours per day for sleep and other personal care leaves 112 hours per week as time endowment (Guerrieri and Lorenzoni, 2017). Hence, average hours worked as a share of the total time endowment gives 36.3% which is targeted in estimation.

Figure 3: Energy expenditure relative to consumption



Note. This figure shows energy expenditure relative to consumption with and without durables for households in consumption expenditure 100 bins. Consumption includes expenses for food, gasoline, rent, utilities, communication, transportation, education, childcare, medical needs, vacations, clothing, and recreational activities. Durable components are car repair expenses, down-, loan-, and lease-payments for vehicle loans as well as other expenditure regarding vehicles. All variables have been adjusted using the OECD equivalence scale and are expressed in 2010-\$.

Wealth-to-income ratio To consider the distribution of endogenous variables in my model, I follow [Stoltenberg and Uhlenborff \(2022\)](#) and target two moments of the wealth-to-income distribution: the 10th percentile as well as the median. A 10th and 50th quantile regression on a constant yields $\hat{\beta}_{10} = -0.253$ and $\hat{\beta}_{50} = 1.056$, respectively. Both values are precisely estimated.

Dirty good allocation rule In the data I only observe expenditures, that is, the product of price and quantity. Hence, the data counterpart to Equation (10) is

$$p_d d_{it} = \delta_0 + \delta_1 e_{it} + \mathbf{X}'_{it} \omega + \varepsilon_{it}, \quad (11)$$

where now e_{it} denotes *observed* total expenditure, $p_d d_{it}$ *observed* expenditure on dirty goods of household i at time t . \mathbf{X} is a vector of controls including household-size dummies, household head's five-year age bracket, region, household and year dummies ([Pedroni, Singh and Stoltenberg, 2022](#); [Straub, 2019](#)). Total expenditure will be instrumented by total income. Lastly, to obtain a proper mapping between arbitrary units in the model and data variables in 2010-\$ units, I scale all data variables with the average dirty goods expenditure, $D^{avg} = p_d \bar{d}$.

Table 1 shows estimation results for the baseline specification (1) and various robustness exercises (2)-(4). In all specifications, the first stage F-statistic is well above 10.¹⁵

To interpret the coefficients, let us look at column (1). The coefficient on total expenditure is equal to 0.0646. According to Equation (10), this coefficient identifies $1 - \eta$, which suggests an η of 0.9354. Second, δ_0 identifies $\eta \frac{p_d d_{it}}{p_d D^{avg}} = \eta \frac{d_{it}}{D^{avg}}$. This constant as well as δ_1 will be targeted in

¹⁵ This rule of thumb is valid as I only consider one endogenous regressor and one instrument ([Stock and Yogo, 2005](#)).

Table 1: Dirty good regression

	dirty goods expenditure			
	(1)	(2)	(3)	(4)
Total expenditure	0.0646 (0.0152)	0.0643 (0.0152)	0.0623 (0.0156)	0.0486 (0.0113)
Constant	0.3571 (0.1004)	0.3517 (0.0743)	0.3682 (0.1016)	0.4169 (0.0896)
Observations	22033	22033	22033	22027
R-squared	0.3059	0.2996	0.3077	0.3100

Note. This table shows second stage (IV) coefficients δ_1 and δ_0 of Equation (11) for different specifications. Column (1) is the baseline case as specified in the text. Column (2) omits the region dummies in the control vector. Column (3) additionally controls for liquid assets. Column (4) uses an alternative measure for consumption as the (instrumented) regressor. Standard errors are corrected for heteroskedasticity and clustered at the household level.

estimation.

3.3 Calibration

To carry out the quantitative exercise, I have to choose functional forms and parameter values, which I now describe. Table 2 summarizes the parameter values.

Preferences I choose a standard value for relative risk aversion, $\gamma = 2$, and set ϵ to target an average Frisch elasticity of labor supply is one. Equation (9) shows the functional form.

Labor productivity process I model the idiosyncratic productivity process as the sum of a persistent and a transitory shock (plus measurement error):

$$\begin{aligned}\log(\theta_{it}) &= \kappa_{it} + \psi_{it} + v_{it} \\ \kappa_{it} &= \rho\kappa_{it-1} + \varepsilon_{it}^{\kappa}.\end{aligned}$$

In particular, the persistence process κ is modelled as an AR(1) with persistence ρ and variance of its innovation of $\sigma_{\varepsilon^{\kappa}}^2$; the transitory shocks ψ are independently and identically distributed with zero mean and variance σ_{ψ}^2 ; v denotes (classical) measurement error with σ_v^2 .

I determine the (annual) variances of the parameters using pre-tax wage residuals estimated from PSID data between 2000 and 2006, following the strategy by Floden and Lindé (2001), and translate them into the 5-year period unit of the model. Moreover, I follow Heathcote, Storesletten and Violante (2010) and Straub (2019) and set $\sigma_v^2 = 0.02$ as estimated in French (2004) to identify the (annual) transitory shock. In Appendix B.2, I describe the estimation procedure in detail.

Final goods production The technology \tilde{F}_1 is assumed to be of the constant elasticity of substitution (CES) form

$$\left[(1-s)(K_1^{\alpha}L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}} \quad (12)$$

with λ as the elasticity of substitution between the capital-labor bundle and energy, and s a share parameter.¹⁶ In equilibrium, the factors of production are rented at rates $r + \delta$, w , and $p_d + \tau_d$, such that by Euler's theorem: $Y = (r + \delta)K_1 + wL_1 + (p_d + \tau_d)E^p$, where δ denotes capital depreciation.

I fix the gross capital share in production α at 0.36 based on standard estimates from the literature (Rognlie, 2016) and the elasticity of substitution between the capital-labor composite and energy λ at 0.547 as found in van der Werf (2008). I follow Straub (2019) and set δ to match a capital-to-output ratio of 3.05. The implied wealth to output ratio is 3.8, close to the most recent estimate of 4 in Piketty and Zucman (2014, Figure IV) for the US.

During estimation I do not impose asset market clearing, hence, I have to specify the calibration of equilibrium prices for this exercise. I fix an annual interest rate at 3%, slightly below the estimate of Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019) for the post-1980 period. Moreover, I set X to normalize the wage rate to 1 and s to match an energy share in production of 5%. Lastly, I set $L = N \sum_s \theta_s f(\theta_s)$, where $f(\cdot)$ denotes the invariant productivity type distribution induced by the Markov chain and where I set $N = 0.33$ since I target this number in estimation.

Energy production Energy is produced using a Cobb-Douglas technology in capital and labor:

$$E = K_2^{\alpha_E} L_2^{1-\alpha_E}. \quad (13)$$

I take $\alpha_E = 0.597$ following Barrage (2020). Moreover, the abatement cost function is

$$\Psi(\mu) = a_1 \mu^{a_2} \quad (14)$$

I follow DICE and set a_2 to 2.6. Hence, the cost function is convex in μ , implying that marginal costs are increasing in abatement. I follow Douenne et al. (2022) and set the scale parameter a_1 to match the backstop price to GDP ratio as in DICE 2016.

Government I follow Fried et al. (2018) and use the three parameter functional form by Gouveia and Strauss (1994) to model the labor income tax function:

$$T(y^{pre}) = \tau_0 \left(y^{pre} - ((y^{pre})^{-\tau_1} + \tau_2)^{-1/\tau_1} \right) \quad (15)$$

Gouveia and Strauss (1994) report $\tau_0 = 0.258$ and $\tau_1 = 0.768$ for the year 1989 - their most recent estimate. Finally, since my calibration is closely related to Fried et al. (2018) I take their estimated value of the parameter $\tau_2 = 1.74$. I set the capital income tax τ^k to 0.36 as in Trabandt and Uhlig (2011).

Climate sector Since I normalize yearly output to unity using X the concrete calibration regarding the damage function and the climate cycle does not change estimation in the initial steady state, for I can update \tilde{X} such that $X = (1 - \mathfrak{D}(S))\tilde{X}$. For completeness, however, I also now describe how I model the climate sector of the economy, which - in the spirit of Nordhaus's DICE model

¹⁶ van der Werf (2008) writes that "the (KL)E nesting structure, that is a nesting structure in which capital and labour are combined first, fits the data best, but we generally cannot reject that the production function has all inputs in one CES function". Another recent example where this particular nesting structure is used is Hassler, Krusell and Olovsson (2021).

Table 2: Preset parameters for estimation

Description	Value	Target/source	
<i>Preferences</i>			
γ	Risk aversion	2.0	literature
ϵ	Curvature of utility from leisure	4.06	Average Frisch elasticity of unity
<i>Productivities (annual)</i>			
ρ	Productivity shock persistence	0.9327	PSID
$\sigma_{\epsilon^k}^2$	Variance of innovations to persistent shock	0.0426	PSID
σ_{ψ}^2	Variance of transitory shocks	0.0507	PSID
σ_v^2	Variance of measurement error	0.02	French (2004, p.608, Table 5)
<i>Production</i>			
<i>Final goods production</i>			
λ	Substitution elasticity	0.547	van der Werf (2008, p.2972, Table 3)
α	Capital share	0.36	literature
δ	Depreciation (annual)	0.133	$K/Y = 3.05$ (FRED)
X	Net total factor productivity	1.468	normalize yearly output to unity
s	Share parameter	0.0277	Energy share in production of 5%
<i>Energy production</i>			
α_2	Capital share	0.597	Barrage (2020)
<i>Abatement</i>			
a_1	Scale abatement cost function	0.630	Backstop price to GDP (see text)
a_2	Exponent abatement cost function	2.6	DICE 2016
<i>Government</i>			
τ_0	Average labor income tax	0.258	Gouveia and Strauss (1994, p.323, Table 1)
τ_1	Progressivity of labor income tax	0.768	Gouveia and Strauss (1994, p.323, Table 1)
τ_2	Scaling parameter	1.74	Fried et al. (2018, p.35, Table 1)
τ^k	Capital income tax	0.36	Trabandt and Uhlig (2011, p.311, Table 1)
B/Y	Public debt / GDP	0.73	FRED
<i>Climate sector</i>			
<i>Damages</i>			
ξ	Damage parameter	0.0015	GDP loss of 5% under BAU
<i>Carbon cycle</i>			
φ	Emissions decay parameter	$1 - \exp(\log(0.5)/60)$	Golosov et al. (2014)
φ_0	Emissions share parameter	$0.5/((1 - \varphi)^6)$	Golosov et al. (2014)

Note. This table shows preset and calibrated parameters of the quantitative model which is used to estimate the remaining parameters via indirect inference. FRED datasources can be found in Appendix B.1.

Table 3: Targeted moments and estimated parameters

Moment	Estimated Parameters	Description
Wealth-to-income ratio - 10 th percentile	\underline{a}	Borrowing limit
Wealth-to-income ratio - Median	β	Discount factor
Expenditure regression (IV), δ_1	η	Clean good elasticity
Expenditure regression (IV), δ_0	\underline{d}	Subsistence level
Average hours worked	χ	Disutility of labor

Note. This table shows targeted moments and estimated parameters using indirect inference.

(Nordhaus, 1992, 1993) - follows Golosov et al. (2014).

Carbon cycle To calibrate φ and φ_0 I follow Golosov et al. (2014).¹⁷ φ is set to capture the fact that excess carbon has a mean-lifetime of about 300 years such that $(1 - \varphi)^{60} = 0.5$, while the calibration for φ_0 captures that half of the CO₂ emissions into the atmosphere are removed after 30 years: $\varphi_0 = \frac{0.5}{(1-\varphi)^6}$

Damage function The functional form for the damage function is taken from Golosov et al. (2014):

$$1 - \mathfrak{D}(S) = e^{-\zeta S_t}, \quad (16)$$

where ζ governs the strength of output damages of a marginal increase in atmospheric carbon.¹⁸ Later, I set the parameter ζ such that in the initial steady state without carbon taxes, damages imply a total loss of 5% of GDP.

3.4 Estimation with indirect inference

The remaining structural parameters to estimate are (i) the utility elasticity η , (ii) the subsistence level \underline{d} , (iii) the disutility of labor χ , (iv) the borrowing limit \underline{a} , and (v) the discount factor β .

The structural parameters of the model will be *jointly* estimated using indirect inference in a just identified system as shown in Table 3. That is, as a starting point I will not target the average expenditure share in energy to better determine which parameter identifies which moment.

Objective function Denote the parameter vector by $\Theta = (\eta, \underline{d}, \chi, \underline{a}, \beta)$ and the vector of moments (as a function of the parameters) by $\mathcal{M} = \left(\delta_0(\Theta), \delta_1(\Theta), \frac{a}{y} \Big|_{50}(\Theta), \frac{a}{y} \Big|_{10}(\Theta), \bar{N}(\Theta) \right)$. Then define the vector of percentage deviations as

$$\mathcal{D} = \left(\mathcal{M} - \widehat{\mathcal{M}} \right) \oslash \widehat{\mathcal{M}}, \quad (17)$$

¹⁷ Golosov and co-authors, in turn, cite Archer (2005) and the 2007 technical summary of the IPCC report (IPCC, 2007)

¹⁸ As Golosov et al. (2014) explain, Equation (16) is an approximation that conflates the *concave* relationship between CO₂ concentrations and temperature, and a *convex* relationship between temperature and damages. In particular, it implies constant marginal damages - measured as a share of GDP: $\frac{\partial Y}{\partial S} = -\zeta$

Table 4: Estimation results

<i>Parameter estimates</i>					
	\underline{d}	η	χ	β	\underline{a}
	0.158 (0.0442)	0.9354 (0.0165)	0.1375 (0.0061)	0.8521 (0.0012)	-1.1202 (0.1194)
<i>Auxiliary model and other moments</i>					
	δ_0	δ_1	\bar{N}	$\frac{\underline{a}}{\underline{y}} _{10}$	$\frac{\underline{a}}{\underline{y}} _{50}$
Data	0.3571	0.0646	0.3627	-0.2527	1.0551
Model	0.3567	0.0646	0.3621	-0.2528	1.0539

Note. Asymptotic standard errors in parentheses are computed using a non-parametric panel bootstrap with 200 repetitions; see Appendix C for details.

where $\widehat{\mathcal{M}}$ denotes the data counterpart of the moment conditions, and \oslash denotes element-wise division. The objective function is then

$$\Theta^* = \arg \min_{\Theta} \mathcal{D}'W\mathcal{D}, \quad (18)$$

where W is a positive definite weighting matrix. In the benchmark estimation, I use the identity matrix as weighting matrix.

The objective function is minimized using a multistart global optimization algorithm ("Tik-Tak"). For local minimization routines in this algorithm, I use the derivative-free BOBYQA routine from starting points that are determined using the Sobol sequence and a pre-testing phase (Guvenen, 2011; Arnoud, Guvenen and Kleineberg, 2019).

4 Estimation results

4.1 Parameter estimates

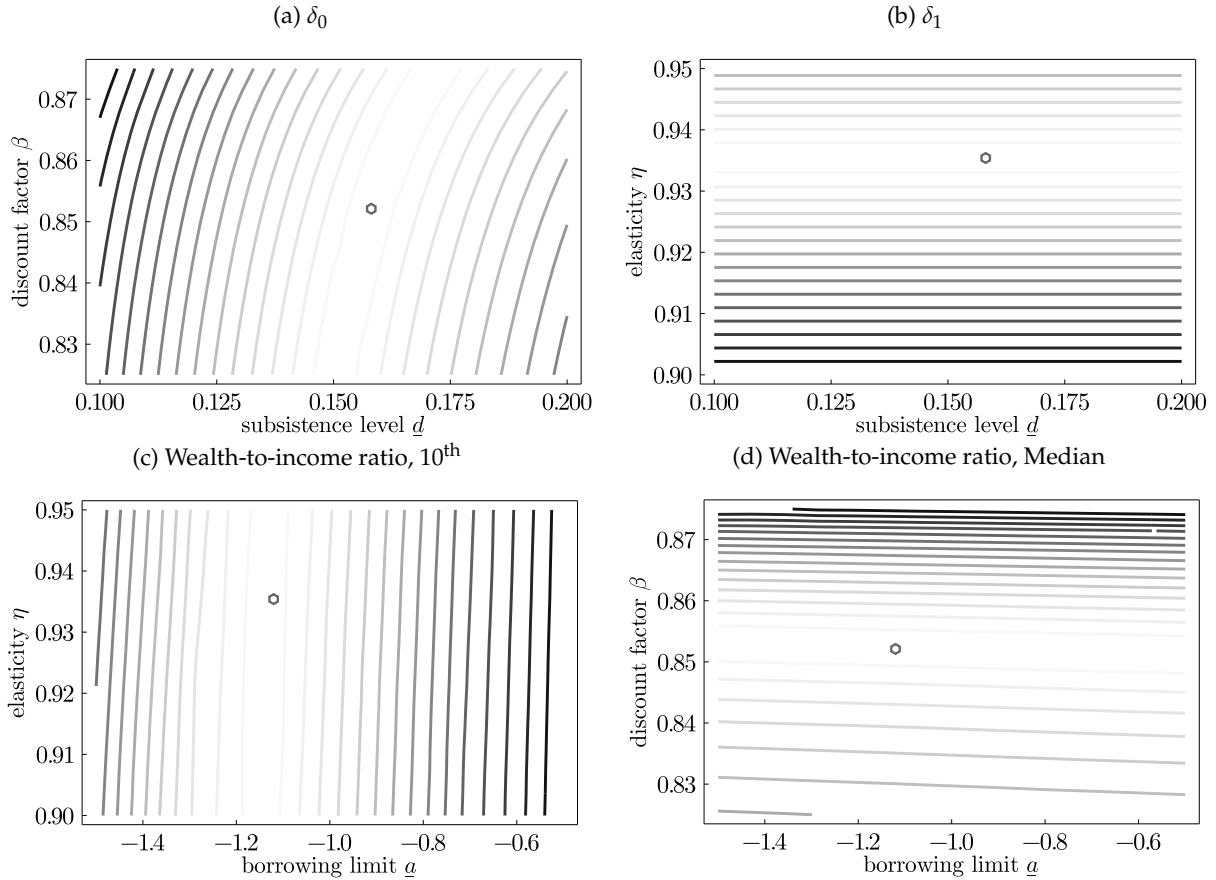
Table 4 shows the estimation results. To gauge precision of my estimates, I construct asymptotic standard errors using a non-parametric bootstrap with 200 repetitions (Appendix C).

Regarding the utility parameters, I estimate $\eta = 0.9354$ and $\underline{d} = 0.158$. Both parameters are precisely estimated. η is comparable with values in the literature, whereas \underline{d} is relatively larger (Fried et al., 2018).¹⁹ Both coefficients of the auxiliary regression are close to their respective model counterpart.

Average hours worked, as a fraction of agents' time endowment, is pinned down well by the disutility of labor χ , which is estimated to equal 0.1375. Again, this parameter is estimated precisely.

¹⁹ Different calibrations, of course, give rise to different subsistence levels of dirty goods consumption. Hence, when comparing \underline{d} to the literature, I compute expenditure on subsistence consumption, $p_a \underline{d}$.

Figure 4: Numerical identification



Note. Absolute deviation of moments implied by the structural model from their data counterparts when parameters on the respective axis are varying. Other parameters are fixed at their estimated value. Darker regions indicate higher deviation.

Given an intertemporal substitution elasticity of $1/\gamma = 1/2$, the discount factor β is relatively low reflecting the 5-year time period of the model. The borrowing limit is $\underline{a} = -1.1202$, which amounts to 22% of average (annual) income can be borrowed every period. The model matches the two targeted moments of the wealth-to-income distribution well.

4.2 Identification

As common in these type of models, I have no proof of global identification of my parameters. However, I want to mention two points that indicate proper identification of the five parameters. First, I assess how different model-implied moments are affected when I change two of the five parameters and fix the remaining three at their best fit value.

Contour plots indicate that the utility parameters are identified in two steps, as indicated by the decision rule in the static model. The relative preference parameter is identified by the model-implied estimate for δ_1 . Conditional on this value, δ_0 identifies the subsistence parameter of consumption. Similar arguments hold for the identification of the discount factor and the borrowing

limit.

Second, after searching a grid of potential values in my optimization algorithm I choose the best 10% and start a local search step from these points. This search procedure converges to the same best fit values for various starting points,²⁰ which suggests that the model is globally identified.

5 The computational experiment

Social welfare function I assume that the social planner is utilitarian and maximizes social welfare defined over households' value functions in stationary equilibrium:

$$SW = \int_{(A \times \Theta)} V_{(\tau_d)}(a, \theta) d\Lambda_{(\tau_d)}. \quad (19)$$

The planner chooses τ_d to maximize Equation (19) while setting g to balance the government budget. Hence, the subscript (τ_d) stresses that the value function and invariant distribution are associated with this particular carbon tax.

The role of precautionary savings To study the role of precautionary savings, I compare the benchmark model to two counterfactuals. First, I compare my results to a complete markets (CM) version of the model in which I get rid of idiosyncratic risk and replace the no-borrowing constraint by the natural debt limit. Since the wealth distribution is indeterminate in the steady-state of this economy (Chatterjee, 1994), I follow Dyrda and Pedroni (2022) and impose the invariate distribution to be the same as in the incomplete markets model (IM) under the optimal tax.

Second, I maximize social welfare for different values of the relative risk aversion parameter, γ . In this way, I induce variation in the strength of the precautionary saving motive of agents.

6 Results

The optimal carbon tax equals $\tau_d^* = 0.391$, about 40% of the energy price. Table 5 shows its impact on equilibrium allocations.

We see that households substitute away from dirty goods and increase their consumption in clean goods. Moreover, average hours worked increases. Note, however, that labor efficiency units decrease, which suggests that labor supply shifts from more to less productive households. In fact, all three production inputs decrease. Nevertheless, overall output increases, because changes on the input side are dwarfed by reductions in environmental damages.

Table 6 shows the result of my first computational exercise. I find that the carbon tax (as a fraction of the price of the dirty good) is more than five times higher under the precautionary saving case. Under this third-best policy setting, that is, keeping other taxes fixed, the main quantitative mechanism is that the carbon tax generates revenue which is recycled lump-sum and provides insurance for households.

²⁰ By construction, the algorithm with every new local search puts more weight on the current local minimum when setting the new starting value. However, it also converges to the same values for early starting values.

Table 5: Changes in aggregate variables with optimal carbon tax

Variables	Percent change
Clean consumption, C	14.50
Dirty consumption, D	-13.00
Hours worked, N	9.32
Labor, L	-1.18
Capital, K	-2.46
Energy, E	-16.78
Output, Y	1.64
Transfers, g	19.85
Emissions, S	-45.15
Damages, D	-44.52

Note. This table compares the changes in aggregate variables of the benchmark economy with and without the tax.

Table 6: Optimal carbon tax with and without precautionary savings

	PS	No PS
τ_d/p_d	0.397	0.07
g/Y	0.20	0.17

Note. This table compares the optimal carbon tax (as a fraction of the price the dirty good) under the benchmark case with precautionary savings (PS) with the complete markets case without precautionary savings (no PS). In the second line, the table depicts the lump-sum transfer to GDP ratio under both cases.

Table 7 shows the result of my second computational exercise. We see that an increase of the curvature of the consumption function by increasing γ - and thus the precautionary motive - the carbon tax decreases.²¹

Table 7: Optimal carbon tax under different values for γ

	$\gamma = 1.5$	$\gamma = 2.0$	$\gamma = 2.5$
τ_d/p_d	0.428	0.397	0.370
g/Y	0.206	0.200	0.196

Note. This table compares the optimal carbon tax (as a fraction of the price the dirty good) for different values of the risk aversion parameter, γ . In the second line, the table depicts the lump-sum transfer to GDP ratio.

7 Conclusion

In this paper, I studied the optimal carbon tax in a climate-economy model with idiosyncratic risk and borrowing constraints in general equilibrium. I first calibrated and estimated the model on U.S. household panel data. Thereby, I proposed a novel strategy to identify the subsistence level of dirty pollution-intensive good consumption using indirect inference. In a next step, I

²¹ In particular, changing γ increases the level of absolute prudence in the consumption composite, which is given by $\frac{1-\gamma}{\bar{c}}$.

used the model as a laboratory to compare the optimal carbon tax in an environment with and without precautionary savings. As my main result, I find that when recycling carbon-tax revenue lump-sum and keeping other policy instruments fixed, precautionary saving increases the optimal carbon tax.

Future research should investigate the implications of precautionary household behavior when optimizing over the full tax mix. That is, the government should also set labor income and capital income taxes which can raise revenue and/or redistribute resources as well. At this stage, all of these tasks fall on to the carbon tax.

Lastly, I find two more avenues of future research of interest. First, the novelty in this paper was the introduction of idiosyncratic risk. Another interesting direction would be to study the distributional consequences of *aggregate* climate uncertainty. This is in particular the case if there is heterogeneous incidence of pollution damages, for instance, due to different abilities in adaptation.

Second, political support for carbon taxation or other forms of corrective pricing has so far been weak.²² This is also partly due to distributional concerns, as the yellow-vest movements in France or Canada demonstrate (Douenne and Fabre, 2022). Thus, an exploration of household heterogeneity in a political climate-economy would be worthy of future research.

²² As of 2020, only 18% of global CO₂ emissions are internalized (Ritchie and Rosado, 2022).

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Appendix

A Model - Details

A.1 Households

I repeat the recursive household problem for ease of exposition.

$$\begin{aligned}
 V(a, \theta) &= \max_{c, d, n, a'} u(c, d, n) + \beta \mathbb{E}_\theta V(a', \theta') \\
 &\text{subject to} \\
 c + (p_d + (1 - \mu)\omega\tau_d)d + a' &\leq (1 + r(1 - \tau^k))a + \underbrace{w\theta n - T^y(w\theta n)}_{\mathcal{T}(w\theta n)} + g \\
 a' &\geq \underline{a} \quad n \geq 0
 \end{aligned}$$

Defining $r(1 - \tau^k) \equiv \tilde{r}$ and $p_d + (1 - \mu)\omega\tau_d \equiv \tilde{p}_d$, the Bellman equation is

$$V(a, \theta) = \max_{c, d, n, a'} u(c, d, n) + \beta \mathbb{E}_\theta V(a', \theta') - \pi^1 \left(c + \tilde{p}_d d + a' - (1 + \tilde{r})a - \mathcal{T}(w\theta n) - g \right) - \pi^2 \left(\underline{a} - a' \right) + \pi^3 n,$$

where π^1, π^2, π^3 denote the Lagrange multipliers on the budget, borrowing, and non-negativity constraint, respectively.

In the following, I use the common notation that $\frac{\partial u(c, d, n)}{\partial c} \equiv u_c$. The first-order conditions of the household are

$$\begin{aligned}
 [c] : \quad u_c &= \pi^1 \\
 [d] : \quad u_d &= \tilde{p}_d \pi^1 \\
 [n] : \quad u_n &= -\pi^1 \mathcal{T}_n(w\theta n) w\theta - \pi^3 \\
 [a'] : \quad \beta \mathbb{E}_\theta [V_a(a', \theta')] &= \pi^1 - \pi^2
 \end{aligned}$$

Substituting out the multiplier π^1 , assuming an interior solution for labor ($\pi^3 = 0$), and using the Envelope condition $V_a(a, \theta) = \pi^1(1 + \tilde{r})$ we get

$$u_c \tilde{p}_d = u_d \tag{A.20}$$

$$u_n = -u_c \mathcal{T}_n(w\theta n) w\theta \tag{A.21}$$

$$u_c \geq \beta(1 + \tilde{r}) \mathbb{E}_\theta [u_{c'}] \tag{A.22}$$

Equation (A.20) is the *intra-temporal* first-order condition between the two consumption goods. Re-arranging yields that in the optimum, the marginal rate of substitution (MRS) between the clean and the dirty good, u_c/u_d , equals the relative price of the clean good in terms of the dirty good, $1/\tilde{p}_d$. Note that the marginal rate of transformation (MRT) between the two goods is $1/p_d$, hence, a positive carbon tax distorts the social optimal goods allocation.

Equation (A.21) is the *intra-temporal* first-order condition between clean consumption and la-

bor. Similar arguments regarding the MRS and MRT as above apply. One could have stated the condition in terms of the dirty good. Again, a positive carbon tax (in addition to the labor tax) distorts the labor supply decision of the household, as it makes leisure cheaper relative to the dirty good.

Equation (A.22) is the *inter-temporal* first-order condition and is the familiar Euler equation. When the borrowing constraint is not binding, π^2 is zero and the equation holds with equality. Here, the capital income tax drives a wedge between MRS ($\frac{u_c}{\beta u_d}$) and MRT ($1 + r$).

A.2 Firms

Energy producer The energy producer maximizes profits by choosing capital, K_2 , labor L_2 , and the fraction of abatement μ under perfect competition using a constant returns to scale technology. It takes prices (r, w, p_d) , pass-through opportunities ω , as well as policy variables as given and obtains zero profits in equilibrium:

$$\max_{K_2, L_2, \mu} \{p_d F_2(K_2, L_2) - (1 - \mu)(1 - \omega)\tau_d F_2(K_2, L_2) - (r + \delta)K_2 - wL_2 - \Psi(\mu)F_2(K_2, L_2)\},$$

where I already substituted in the technology constraint $E = \cdot F_2(K_2, L_2)$, adjusted for the 5-year model period.

The first-order conditions are

$$\begin{aligned} [p_d - (1 - \mu)(1 - \omega)\tau_d + \Psi(\mu)]F_{2,K_2} &= r + \delta \\ [p_d - (1 - \mu)(1 - \omega)\tau_d + \Psi(\mu)]F_{2,L_2} &= w \\ \tau_d(1 - \omega) &= \Psi'(\mu) \end{aligned}$$

Final good producer The final good firm maximizes its profits by choosing capital, K_1 , labor L_1 , and energy E^p under perfect competition using a constant returns to scale technology. Hence, it takes prices (r, w, p_d) , policy variables as well as carbon-tax pass-through of the energy producer as given and obtains zero profits in equilibrium:

$$\max_{K_1, L_1, E} \{F(K, E, L; X, S) - (r + \delta)K_1 - wL_1 - (p_d + (1 - \mu)\omega\tau_d)E^p\}$$

Moreover, the firm compensates households for depreciation, δ .

The first-order condition of the firm implied by profit maximization - substituting in the CES production function from the main text - are

$$\begin{aligned} p_d + (1 - \mu)\omega\tau_d &= \frac{\partial F_1}{\partial E^p} = X \left[(1 - s)(K_1^\alpha L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{1}{\lambda-1}} s(E^p)^{-\frac{1}{\lambda}} \\ w &= \frac{\partial F_1}{\partial L_1} = X \left[(1 - s)(K_1^\alpha L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{1}{\lambda-1}} (1 - s)(K_1^\alpha L_1^{1-\alpha})^{\frac{-1}{\lambda}} (1 - \alpha)K_1^\alpha L_1^{-\alpha} \\ r + \delta &= \frac{\partial F_1}{\partial K_1} = X \left[(1 - s)(K_1^\alpha L_1^{1-\alpha})^{\frac{\lambda-1}{\lambda}} + s(E^p)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{1}{\lambda-1}} (1 - s)(K_1^\alpha L_1^{1-\alpha})^{\frac{-1}{\lambda}} \alpha K_1^{\alpha-1} L_1^{1-\alpha} \end{aligned}$$

As usual, the prices of the inputs are equal to their marginal products.

A.3 Goods market clearing

To avoid notational clutter, I derive the goods-market clearing as if the time period was a year. Aggregate the household budget constraint over household and impose asset market clearing $A = B + K$ in the steady-state:

$$C + (p_d + (1 - \mu)\omega\tau_d)D + (B + K) = (1 + (1 - \tau^k)r)(B + K) + wL - \int T^y d\Lambda + g.$$

Rewrite the government budget constraint as $g - \int T^y d\Lambda = \tau^k r(K + B) + \tau_d(1 - \mu)E - rB$ ²³, plug it in the aggregated household constraint above and collect terms:

$$C + (p_d + (1 - \mu)\omega\tau_d)D = rK + wL + \tau_d(1 - \mu)E.$$

Extend with δK and $(p_d + (1 - \mu)\omega\tau_d)E^p$, recall that $E^p + D, K_1 + K_2 = K, L_1 + L_2 = L$, and use Euler's theorem to obtain:

$$\begin{aligned} C + \delta K &= \underbrace{(r + \delta)K_1 + wL_1 + (p_d + (1 - \mu)\omega\tau_d)E^p}_Y \\ &\quad + \tau_d(1 - \mu)E - (p_d + (1 - \mu)\omega\tau_d)(E^p + D) + (r + \delta)K_2 + wL_2 \end{aligned}$$

Finally, use the first-order conditions of the energy producer and again use Euler's theorem to write

$$\begin{aligned} C + \delta K &= Y + [p_d - (1 - \mu)\tau_d(1 - \omega) - \Psi(\mu)] \underbrace{(F_{2,K_2}K_2 + F_{2,L_2}L_2)}_E + \tau_d(1 - \mu)E - (p_d + (1 - \mu)\omega\tau_d)E \\ C + \delta K + \Psi(\mu)E &= Y + \underbrace{[p_d - (1 - \mu)\tau_d(1 - \omega)]E + \tau_d(1 - \mu)E - (p_d + (1 - \mu)\omega\tau_d)E}_0 \\ C + \delta K + \Psi(\mu)E &= Y \end{aligned}$$

Hence, the final good can be used for consumption, investment, and abatement.²⁴

B Calibration - Details

B.1 Macroeconomic variables

Capital-output-ratio (K/Y) Current-Cost Net Stock of Fixed Assets (2014) in current dollars divided by Gross Domestic Product (2014) in current dollars; both series from the FRED database with series tags K1TTOTL1ES000 and GDPA, respectively.

Bond-output-ratio (B/Y) Federal Debt Held by the Public (2014) in current dollars divided by Gross Domestic Product (2014) in current dollars; both series from the FRED database with series tags FYGFDPUN and GDPA, respectively.

²³ To be precise, the government receives carbon tax revenue from the household, $\tau_d(1 - \mu)\omega D$, from the final goods firm, $\tau_d(1 - \mu)\omega E^p$, and from the energy producer $\tau_d(1 - \mu)(1 - \omega)E$. These three terms sum up to $\tau_d(1 - \mu)E$.

²⁴ The second term on the left-hand-side equals investment I , since in the steady-state the law of motion of capital $K_{t+1} = (1 - \delta)K_t + I_t$ collapses to $\delta K = I$.

B.2 Estimation of the productivity process

In the following, I explain how I estimate the labor productivity process which I use in my quantitative model.²⁵ I follow [Floden and Lindé \(2001\)](#) and measure productivity as agent's "hourly [pre-tax] wage rate relative to all other agents" (p. 416). The data is taken from the PSID and refers to labor income as described in the main text. The productivity process is estimated on yearly data. To avoid notational clutter, I denote yearly time-steps by τ , compared with a model period (5 years) denoted by t .

As my productivity process, I take the following standard persistent-transitory specification for log wages at year τ

$$\log \hat{\theta}_{i\tau} = f(\mathbf{X}_i, \beta) + \kappa_{i\tau} + \psi_{i\tau} + v_{i\tau} \quad (\text{B.23})$$

$$\kappa_{i\tau} = \rho \kappa_{i\tau-1} + \varepsilon_{i\tau}^{\kappa}, \quad (\text{B.24})$$

where $f(\mathbf{X}_i, \beta)$ denotes a set of individual-specific controls, $\kappa_{i\tau}$ is an AR(1) process with persistence ρ and innovation-variance $\sigma_{\varepsilon^{\kappa}}^2$, $\psi_{i\tau}$ is a transitory component with variance σ_{ψ}^2 , and $v_{i\tau}$ is measurement error.

Given the short time horizon when estimating the productivity process, instead of estimating the household fixed effect directly, I model the permanent component by controlling for individual-specific characteristics, \mathbf{X}_i .

Moreover, the measurement error term cannot be identified from the transitory term. Hence, I follow the literature and set the variance of the measurement error term to 0.02 ([French, 2004](#); [Heathcote et al., 2010](#); [Straub, 2019](#)).

The estimation then proceeds along the following steps. First, I residualize log wages using

$$\log \theta_{i\tau} = \log \hat{\theta}_{i\tau} - f(\mathbf{X}_i, \hat{\beta}).$$

Second, I compute empirical variances and covariances from these residuals and stack them in the vector $\vec{\mathfrak{M}}$. The theoretical variances and covariances, on the other hand, can be computed using

$$\begin{aligned} \text{var}(\log \theta_{i\tau}) &= \frac{\sigma_{\varepsilon^{\kappa}}^2}{1 - \rho^2} + \sigma_{\psi}^2 + \sigma_v^2 \\ \text{cov}(\log \theta_{i\tau}, \log \theta_{i\tau-h}) &= \rho^h \frac{\sigma_{\varepsilon^{\kappa}}^2}{1 - \rho^2}. \end{aligned}$$

I denote the stacked theoretical (co)variances by $\vec{\mathfrak{m}}(\rho, \sigma_{\varepsilon^{\kappa}}^2, \sigma_{\psi}^2)$. This formulation stresses that $\vec{\mathfrak{m}}$ is a function of the parameters that we seek to estimate.

Lastly, I apply a minimum distance estimation (MDE) to minimize the weighted distance, $\mathfrak{L}(\rho, \sigma_{\varepsilon^{\kappa}}^2, \sigma_{\psi}^2) = \vec{\mathfrak{M}} - \vec{\mathfrak{m}}$, between theoretical and empirical moments/covariances:

$$\min_{\rho, \sigma_{\varepsilon^{\kappa}}^2, \sigma_{\psi}^2} \mathfrak{L}(\rho, \sigma_{\varepsilon^{\kappa}}^2, \sigma_{\psi}^2)' \mathfrak{W} \mathfrak{L}(\rho, \sigma_{\varepsilon^{\kappa}}^2, \sigma_{\psi}^2)$$

²⁵ The exposition here follows the one in [Straub \(2019\)](#) who uses a similar strategy to estimate a process for log income and from whose description I learned a lot.

Table B.8: Estimated parameters - Productivity process

ρ	$\sigma_{\varepsilon^k}^2$	σ_{ψ}^2
0.9327	0.0426	0.0507
(0.0092)	(0.0060)	(0.0049)

Note. This table shows the estimated parameters of the productivity process. Standard errors are bootstrapped using a non-parametric block bootstrap at the household level with 500 iterations.

As is standard in this procedure, I use the identity matrix as weighting matrix \mathfrak{W} which was shown to be more robust to small sample bias (Altonji and Segal, 1996).

Table B.8 shows the result of the MDE. Standard errors are obtained by using a non-parametric block bootstrap (Cameron and Trivdei, 2005, p.362/p.377).

5-year time period To translate these values that were estimated on annual data to their 5-year model counterparts, I proceed in two steps: First, I iterate the persistent component backward such that

$$\kappa_{i\tau} = \rho^5 \kappa_{i\tau-5} + \underbrace{\sum_{s=0}^4 \rho^s \varepsilon_{i\tau-s}^k}_{\tilde{\varepsilon}_{i\tau}^k} \quad (\text{B.25})$$

I can compute the variance of $\tilde{\varepsilon}_{i\tau}^k$ given the annual estimate:

$$\sigma_{\tilde{\varepsilon}^k}^2 = \text{var}(\tilde{\varepsilon}_{i\tau}^k) = \sum_{s=0}^4 \rho^{2s} \text{var}(\varepsilon_{i\tau-s}^k) = \sum_{s=0}^4 \rho^{2s} \sigma_{\varepsilon^k}^2 \quad (\text{B.26})$$

Second, I set σ_{ψ}^2 such that its relative contribution to the variance of labor productivity is the same as under the annual estimates. In other words,

$$\frac{\sigma_{\tilde{\varepsilon}^k}^2}{\sigma_{\psi}^2} = \frac{\sigma_{\varepsilon^k}^2}{\sigma_{\psi}^2} \quad (\text{B.27})$$

C Estimation - Details

Standard errors [Gourieroux et al. \(1993\)](#) show that under no observable exogenous variables that enter the moments, the asymptotic variance-covariance matrix is

$$\text{COV} = \left(1 + \frac{1}{B}\right) \left[\frac{\partial \mathcal{M}^{*\top}}{\partial \Theta} W \frac{\partial \mathcal{M}^*}{\partial \Theta} \right]^{-1} \frac{\partial \mathcal{M}^{*\top}}{\partial \Theta} W \text{cov}(\mathcal{M}^B) W \frac{\partial \mathcal{M}^*}{\partial \Theta} \left[\frac{\partial \mathcal{M}^{*\top}}{\partial \Theta} W \frac{\partial \mathcal{M}^*}{\partial \Theta} \right]^{-1}$$

where B denotes the number of bootstrap repetitions, $\mathcal{M}^* = \mathcal{M}(\Theta^*)$ and $\text{cov}(\mathcal{M}^B)$ is the covariance matrix of the bootstrapped moments. I bootstrap standard errors using a non-parametric panel (block) method.

D Additional tables & figures

Table D.1: Summary statistics

Variable	Mean	Std. dev.	Median
<i>Demographics</i>			
Age	41.52	10.12	41
Sex	0.821	0.383	1
Household size	2.919	1.448	3
Number of children	0.990	1.203	1
Married	0.663	0.473	1
<i>Education</i>			
Elementary or middle school	0.0721	0.259	0
Finished high school	0.244	0.430	0
Some college	0.281	0.449	0
Finished college	0.213	0.410	0
Postgrad. qualification	0.189	0.392	0
<i>Region</i>			
Northeast	0.166	0.372	0
Midwest	0.302	0.459	0
South	0.337	0.473	0
West	0.195	0.397	0
Net income	32,080	19,127	28,132
Net labor income	41,119	29,023	34,637
Wealth	93,804	179,069	30,237
Nondurable consumption	14,919	7,714	13,119
Total consumption	17,136	9,502	14,798

Note. This table shows summary statistics regarding demographic and economic variables of the data which is used in estimation.

E Computational appendix

E.1 Computing the household's optimal decision

I use a variant of the endogenous gridpoint method (EGM) to solve the household's decision problem. Compared to the basic version developed by [Carroll \(2006\)](#), my version accommodates two goods and endogenous labor supply with possibly non-linear taxation.

Grids I represent asset positions by discrete points on an exponentially-spaced grid $\mathcal{A} \subset [a, \bar{a}]$, where \bar{a} is chosen large enough such that the upper bound is never binding. I discretize the productivity Markov process with a finite-state Markov chain using [Rouwenhorst \(1995\)](#)'s method.

The inputs for this method, such as the persistence parameter ρ , are obtained in Appendix B.2.

Endogenous gridpoint method

Step 1 I start with a guess of the clean consumption policy function defined on the *future* asset and productivity grid, $c(a', \theta')$. Using the intra-temporal first-order condition between clean and dirty consumption, I can express the dirty consumption policy function $d(a', \theta')$ as a function of $c(a', \theta')$:

$$d(a', \theta') = \frac{1 - \eta}{(p_d + \tau_d)\eta} c(a', \theta') + \underline{d}. \quad (\text{E.28})$$

Step 2 Hence, for each pair (a', θ) where the household is not constrained and the Euler equation (EE) holds with equality, I can solve *analytically* for the value $c(a', \theta)$.²⁶ $c(a', \theta)$ is essentially on the left-hand side of the EE and represents the value of consumption today, which is consistent with having a' assets tomorrow if the productivity shock today is θ :

$$u_c(c(a', \theta)) = \beta(1 + \tilde{r})\mathbb{E}_\theta [u_c(c(a', \theta'))] \quad (\text{E.29})$$

Note that I write u_c explicitly as a function of c only, as the utility is separable in consumption and labor, and d is implied by Equation (E.28).

Step 3 With $c(a', \theta)$ in hand, I can solve for $n(a', \theta)$ using the intra-temporal FOC between clean consumption and labor. In the following, I assume an interior solution:

$$-u_n(n(a', \theta)) = u_c(c(a', \theta))\mathcal{T}_n, \quad (\text{E.30})$$

where \mathcal{T}_n denotes $\frac{\partial \mathcal{T}}{\partial n}$. Under linearity of \mathcal{T} , Equation (E.30) can also be solved analytically for $n(a', \theta)$. Otherwise, a root-finding step has to be implemented at every point in the state space. In the benchmark case, I use a version of Brent's method, modified to take into account multiple roots under the non-linearity of \mathcal{T} as well as the corner solution if $n(a', \theta) = 0$.

Step 4 I can then invert the budget constraint to solve for the value of assets today, $a^*(a', \theta)$, which are consistent with the future assets (on grid) and the choices made above.

$$a^* = \frac{1}{1 + \tilde{r}} \left(c(a', \theta) + (p_d + \tau_d)d(a', \theta) + a' - \mathcal{T}(w\theta n(a', \theta)) \right), \quad (\text{E.31})$$

implying $\tilde{c}(a^*, \theta) = c(a', \theta)$. Note that these a^* are not on the grid (whence the name) and change each iteration. To obtain a new guess for the clean consumption policy function which is defined on the grid, I linearly interpolate on $(a^*, \tilde{c}(a^*, \theta))$ and apply this mapping to the exogenous grid a' . Use the new guess as a starting point in Step 2 above.

I repeat the above iteration procedure until convergence between two successive clean consumption policy functions is achieved: $\|c^{n+1} - c^n\| < 10^{-7}$, where $\|\cdot\|$ denotes the supnorm and n is the iteration counter.

²⁶ Of course, this step depends on the invertibility of the utility function. Other functional forms for the consumption composite might not make this feasible.